Auction Design:
Indirect Contention
Updated ExampleforComplexSet
December2015


Updated

## Updated

## Feasible Sets

C1

B

$$
\underset{\text { or }}{ }+C_{2}
$$

A1 + C1
or
$\mathrm{A}_{1}+\mathrm{C}_{2}$
or
$\mathrm{A}_{2}+\mathrm{C}_{1}$
or
$A 2+C_{2}$

## Updated Complex Example 1

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| 2 |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 1: Enduring Applications:
$A_{1}, A 2$, and $B$ bid the End of Round Price
$C_{1}$ and $C_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\{A 1, C 1\},\{A 1, C 2\},\{A 2, C 1\},\{A 2, C 2\}$ and $\{B, C 2\}$

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 k$ | $\$ 400 k$ | $\$ 400 k$ | $\$ 400 k$ | $\$ 200 k$ | $\$ 100 k$ |
| 2 | $\$ 900 k$ | $\$ 900 k$ | $\$ 800 k$ | $\$ 900 k$ | $\$ 300 k$ | $\$ 200 k$ |
| $\mathbf{3}$ |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| $\mathbf{5}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 2: Enduring Applications: A1, and B met the End of Round Price
$C_{1}$ and $C_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\left\{A_{1}, C_{1}\right\}$, $\left\{A_{1}, C_{2}\right\}$, and $\left\{B, C_{2}\right\}$

Eliminated: A2 did not meet the End of Round Price while A1, positioned the same, bid more

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| $\mathbf{4}$ |  |  |  |  |  |  |
| $\mathbf{5}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 3: Enduring Applications: A1 and B met the End of Round Price
$\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\left\{\mathrm{A}_{1}, \mathrm{C}_{1}\right\}$, $\left\{A_{1}, C_{2}\right\}$ and $\{B, C 2\}$

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| $\mathbf{4}$ | $\$ 2.2 \mathrm{~m}$ | $\$ 2.2 \mathrm{~m}$ | Eliminated | $\$ 1.75 \mathrm{~m}$ | $\$ 500 \mathrm{k}$ | $\$ 400 \mathrm{k}$ |
| $\mathbf{5}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 4: Enduring Applications: A1 met the End of Round Price
$\mathrm{C}_{1}$ did not meet the End of Round Price; however, was in the Feasible Set $\left\{A_{1}, \mathrm{C}_{1}\right\}$
$\mathrm{C}_{2}$ did not meet the End of Round Price; however, was in the Feasible Set $\left\{A_{1}, C_{2}\right\}$
Eliminated Applications: B did not meet the End of Round Price
Winning Application: A1
Winning Payment for A1: $\$ 1.75 \mathrm{~m}$ (the greater of A2's final Exit Bid ( $\$ 800 \mathrm{k}$ ) and B's final Exit Bid

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| $\mathbf{4}$ | $\$ 2.2 \mathrm{~m}$ | $\$ 2.2 \mathrm{~m}$ | Eliminated | $\$ 1.75 \mathrm{~m}$ | $\$ 500 \mathrm{k}$ | $\$ 400 \mathrm{k}$ |
| $\mathbf{5}$ | $\$ 3.0 \mathrm{~m}$ | Prevailed | Eliminated | Eliminated | $\$ 600 \mathrm{k}$ | $\$ 500 \mathrm{k}$ |
| Results |  |  |  |  |  |  |

Round 5: Enduring Applications: None
Eliminated Applications: C $_{2}$ did not meet the End of Round Price and $C_{1}$, positioned the same, bid more

Winning Application: C1
Winning Payment: C1 will pay \$500k (C2's Exit Bid Price)

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| $\mathbf{4}$ | $\$ 2.2 \mathrm{~m}$ | $\$ 2.2 \mathrm{~m}$ | Eliminated | $\$ 1.75 \mathrm{~m}$ | $\$ 500 \mathrm{k}$ | $\$ 400 \mathrm{k}$ |
| $\mathbf{5}$ | $\$ 3.0 \mathrm{~m}$ | Prevailed | Eliminated | Eliminated | $\$ 600 \mathrm{k}$ | $\$ 500 \mathrm{k}$ |
| Results |  | Prevailed | Eliminated | Eliminated | Prevailed | Eliminated |

Results: Winning Payments:
A1 will pay $\$ 1.75 \mathrm{~m}$ (the greater of A2's final Exit Bid (\$80ok) and B's final Exit Bid (\$1.75m))
C1 will pay \$500k (C2's Exit Bid Price)

## Updated Complex Example 2

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| 2 |  |  |  |  |  |  |
| $\mathbf{3}$ |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 1: Enduring Applications:
$A_{1}, A_{2}$, and $B$ bid the End of Round Price
$\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\left\{A_{1}, C_{1}\right\},\left\{A_{1}, C_{2}\right\}\left\{A_{2}, C_{1}\right\},\left\{A_{2}, C_{2}\right\}$ and $\left\{B, C_{2}\right\}$

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 301 \mathrm{k}$ |
| $\mathbf{3}$ |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 2: Enduring Applications: A1 and B met the End of Round Price
Eliminated Applications: A2 did not meet the End of Round Price while A1, positioned the same, bid more;
C1 did not meet the End of Round Price while C2, positioned the same or better, bid more
Winning Application: C2
Winning Payment for C2: \$300k (C1's exit bid)

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 301 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | Eliminated | Prevailed |
| $\mathbf{4}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 3: Enduring Applications: A1 and B met the End of Round Price

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | \$300k | $\$ 301 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | Eliminated | Prevailed |
| $\mathbf{4}$ | $\$ 2.2 \mathrm{~m}$ | $\$ 2.2 \mathrm{~m}$ | Eliminated | $\$ 1.75 \mathrm{~m}$ | Eliminated | Prevailed |
| Results |  |  |  |  |  |  |

Round 4: Enduring Applications: A1 met the End of Round Price
Eliminated Applications: B did not meet the End of Round Price
Winning Application: A1
Winning Payment for A1: $\$ 1.75$ m (the greater of A2's final Exit Bid ( $\$ 800 \mathrm{k}$ ) and B's final Exit Bid (\$1.75m))

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 301 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | Eliminated | Prevailed |
| $\mathbf{4}$ | $\$ 2.2 \mathrm{~m}$ | $\$ 2.2 \mathrm{~m}$ | Eliminated | $\$ 1.75 \mathrm{~m}$ | Eliminated | Prevailed |
| Results |  | Prevailed | Eliminated | Eliminated | Eliminated | Prevailed |

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Results: Winning Payment: A1 will pay \(\$ 1.75 \mathrm{~m}\) (the greater of A2's final Exit Bid ( \(\$ 800 \mathrm{k}\) ) and B's final Exit Bid ( \(\$ 1.75 \mathrm{~m}\) )) C2 will pay \$300k (C1's Exit Bid Price)
```


## Updated Complex Example 3

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| 2 |  |  |  |  |  |  |
| $\mathbf{3}$ |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 1: Enduring Applications:
$A_{1}, A_{2}$, and $B$ bid the End of Round Price
$\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\left\{A_{1}, C_{1}\right\},\left\{A_{1}, C_{2}\right\}\left\{A_{2}, C_{1}\right\},\left\{A_{2}, C_{2}\right\}$ and $\left\{B, C_{2}\right\}$

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 k$ | $\$ 400 k$ | $\$ 400 k$ | $\$ 400 k$ | $\$ 200 k$ | $\$ 100 k$ |
| $\mathbf{2}$ | $\$ 900 k$ | $\$ 900 k$ | $\$ 800 k$ | $\$ 900 k$ | $\$ 300 k$ | $\$ 200 k$ |
| $\mathbf{3}$ |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

## Round 2: Enduring Applications: A1, and B met the End of Round Price $C_{1}$ and $C_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\left\{A_{1}, C_{1}\right\}$, $\left\{A 1, C_{2}\right\}$, and $\left\{B, C_{2}\right\}$

Eliminated: A2 did not meet the End of Round Price while A1, positioned the same, bid more

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| $\mathbf{4}$ |  |  |  |  |  |  |
| Results |  |  |  |  |  |  |

Round 3: Enduring Applications: A1 and B met the End of Round Price
$\mathrm{C}_{1}$ and $\mathrm{C}_{2}$ did not meet the End of Round Price; however, are in a Feasible Set $\left\{\mathrm{A}_{1}, \mathrm{C}_{1}\right\}$, $\left\{A_{1}, C 2\right\}$ and $\{B, C 2\}$

| Round | End of Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | \$400k | \$400k | \$400k | \$400k | \$200k | \$100k |
| 2 | \$900k | \$900k | \$800k | \$900k | \$300k | \$200k |
| 3 | \$1.5m | \$1.5m | Eliminated | \$1.5m | \$400k | \$300k |
| 4 | \$2.2m | \$1.75m | Eliminated | \$2.2m | \$400k | \$300k |
| Results |  |  |  |  |  |  |

Round 4: Enduring Applications: B met the End of Round Price
$C_{2}$ did not meet the End of Round Price, however is part of Feasible Set $\left\{B, C_{2}\right\}$, which did meet the End of Round Price

Eliminated Applications: A1 did not meet the End of Round Price; C1 did not meet the End of Round Price

Winning Applications: B and C2
Winning Payment for B: \$1.892m ((2.2m/(2.2m+300k))*(1.75m+400k))
Winning Payment for C2: \$258k ((300k/(2.2m+300))*(1.75m+400k))

| Round | End of <br> Round Price | A1 | A2 | B | C1 | C2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 400 \mathrm{k}$ | $\$ 200 \mathrm{k}$ | $\$ 100 \mathrm{k}$ |
| $\mathbf{2}$ | $\$ 900 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 800 \mathrm{k}$ | $\$ 900 \mathrm{k}$ | $\$ 300 \mathrm{k}$ | $\$ 200 \mathrm{k}$ |
| $\mathbf{3}$ | $\$ 1.5 \mathrm{~m}$ | $\$ 1.5 \mathrm{~m}$ | Eliminated | $\$ 1.5 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| $\mathbf{4}$ | $\$ 2.2 \mathrm{~m}$ | $\$ 1.75 \mathrm{~m}$ | Eliminated | $\$ 2.2 \mathrm{~m}$ | $\$ 400 \mathrm{k}$ | $\$ 300 \mathrm{k}$ |
| Results |  | Eliminated | Eliminated | Prevailed | Eliminated | Prevailed |

Results: Winning Payments:
B will pay: $\$ 1.892 \mathrm{~m}((2.2 \mathrm{~m} /(2.2 \mathrm{~m}+300 \mathrm{k})) *(1.75 \mathrm{~m}+400 \mathrm{k}))$
C2 will pay: $\$ 258 \mathrm{k}((300 \mathrm{k} /(2.2 \mathrm{~m}+300)) *(1.75 \mathrm{~m}+400 \mathrm{k}))$

